

Fractionalized exciton Fermi surfaces and condensates in two-component quantized Hall states

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A wide variety of two-dimensional electron systems (2DES) allow for independent control of the total and relative charge density of two-component fractional quantum Hall (FQH) states. Intriguingly, in some systems at total filling $\nu_T = \frac{1}{2}$ a transition is observed between a compressible and incompressible phase as the relative charge density is tuned. Naively this transition would be mediated by bosonic excitons, which consist of an electron-hole excitation between the two components. We show that these systems can also support fermionic excitons, due to the topological order of even denominator FQH states. Bosonic excitons can condense to yield interlayer coherent phases, while fermionic excitons can form an emergent Fermi surface in an incompressible FQH insulator. Exact diagonalization studies for up to 16 electrons find that the fermionic exciton is lower in energy, suggesting that a fractionalized “exciton metal” may be realizable experimentally. We suggest several detection schemes by which the different scenarios may be experimentally distinguished.

Introduction – Two-component quantum Hall systems have long been known to host rich phase diagrams, exhibiting intrinsically two-component fractional quantum Hall (FQH) states, broken symmetry states, and quantum phase transitions at fixed total filling fraction ν_T [1–3]. In many experimental systems, the relative filling $\nu_+ - \nu_-$ can be tuned *in situ*. When tunneling between the components is effectively zero, the system acquires an enhanced total and relative $U(1)_T \times U(1)_r$ symmetry due to the independently conserved charges of the two components. This situation is most easily realized when the two components are related by spin or valley symmetry[4–8], or in double layer systems in which a barrier suppresses interlayer tunneling[9, 10].

In this Letter, we consider the phase diagram of two component FQH phases at $\nu_T = \frac{1}{2}$, as has been studied in wide quantum wells[11–14], ZnO heterostructures[8], and, most recently, bilayer graphene (BLG)[15]. We find that the relative $U(1)_r$ symmetry has important consequences for the interpretation of observed transitions between incompressible even denominator FQH and compressible composite fermion liquid phases at half filling. In particular, we investigate how intercomponent excitons, whose density is set by $\nu_+ - \nu_-$, can delocalize into a variety of degenerate quantum liquids. These liquids can form an intermediate phase between the compressible and incompressible limits.

The peculiar nature of even-denominator FQH states guarantees that in addition to the familiar bosonic exciton (b-Exc), the system also hosts a topologically non-trivial fermionic exciton (f-Exc). If the f-Exc is lower in energy, the system naturally forms an “exciton metal” at finite f-Exc density, with insulating charge transport but metallic counterflow resistance. We consider two main situations. First we discuss the crossing of an $N = 0$ and $N = 1$ LL in the limit where the distance d between the two components is small compared to the magnetic length ℓ_B , as occurs in ZnO[8], wide quantum

wells[12], and BLG[15]. Our exact diagonalization calculations show that the f-Exc is indeed lower in energy. In the second scenario, relevant to a bilayer with $d/\ell_B \gtrsim 1$, we consider a crossing of two $N = 0$ levels, where we also argue that the f-Exc will determine the nature of the intermediate phase.

$(N_+, N_-) = (1, 0)$: *Pfaffian exciton metal*. Here we consider the case where the LL crossing is occurring between an $N_+ = 1$ LL in the top layer and an $N_- = 0$ LL in the bottom layer; the filling fraction of the two layers is $\nu_+ = 1/2 - \delta$ and $\nu_- = \delta$. Because $N_+ = 1$, when $\delta = 0$ the system should form an incompressible FQH state in the top layer, analogous to the 5/2-plateau of GaAs[16]; we assume that the system forms a Moore-Read Pfaffian FQH state [17], although any even-denominator FQH state will lead to essentially the same conclusions.[33] When $\delta = 1/2$, the particles reside in an $N_- = 0$ level, so the system forms a composite Fermi liquid in the limit of weak disorder [18–20].

What is the fate of the system at intermediate δ ? As δ increases from zero, the top layer loses charge to the bottom layer. Due to the strong Coulomb interaction between layers, excitons will form, with $-e$ charge in the top layer and e charge in the bottom layer, with a binding energy on the order of the interlayer Coulomb interaction. Crucially, at $\nu_T = \frac{1}{2}$ this system supports two topologically distinct types of excitons. The bosonic exciton (b-Exc) is formed when an electron is transferred from the Pfaffian state in the top layer to the bottom layer. On the other hand, the Pfaffian state also has a charge $-e$ bosonic excitation, which can be thought of as a Laughlin quasiparticle associated with inserting two flux quanta into the system. A bound state of the charge $-e$ boson in the top Pfaffian layer and an electron is a *fermionic* exciton (f-Exc). In contrast to the b-Exc, the f-Exc is a topologically non-trivial quasiparticle, as it is a bound state of the b-Exc and the anyonic ‘neutral fermion’ ψ_{NF} of the Pfaffian phase. As we will demonstrate within the long wavelength effective

field theory, this f-Exc is in fact coupled to an emergent Z_2 gauge field; a pair of f-Exc's is topologically equivalent to a pair of b-Exc's.

Because excitons are neutral particles, they have some non-zero dispersion $\epsilon(k)$ and can delocalize. If the excitons attract, there may be an instability and the transition will be discontinuous, but otherwise we can consider three types of ground states for the exciton system: density-wave, condensate, and metal. First, depending on the residual interactions between the excitons, it may be preferable for them to form a density-wave state, for example stripes or a Wigner crystal of excitons. In the presence of weak disorder that pins the density wave, this state can be viewed as a localized state of excitons, and can be thought of as a Bose glass or Anderson insulator for the b-Exc, f-Exc respectively.

As the density of excitons increases with δ , the b-Exc can potentially undergo a quantum phase transition to a superfluid, spontaneously breaking $U(1)_r$. Analogous to $\nu_T = 1$ [2], the condensation of the b-Exc leads to an interlayer coherent Moore-Read Pfaffian state. Alternatively, if the f-Exc are more stable, increasing their density leads to a Fermi surface whose volume is set by δ . In this case, the Pfaffian state coexists with a Fermi surface of f-Exc's, leading to insulating charge transport but metallic counterflow. There is no sharp transition between the Anderson insulator state and the "metallic" state of excitons, because in two dimensions all states are localized by disorder. At finite temperature there is a crossover from the localized to delocalized regime as the temperature is increased, with a crossover temperature $T^* \sim e^{-\epsilon_F/W}$, where ϵ_F is the Fermi energy and W is the disorder strength.

Let us now describe the above scenario more concretely in terms of a long wavelength effective field theory. c_+ and c_- denote the electrons in the two layers. To describe the system at $\nu_T = 1/2$, attach two flux quanta to each electron, to obtain composite fermions ψ_+ and ψ_- . It is convenient to describe this in terms of a parton construction (see e.g. [21]) $c_\pm = b\psi_\pm$, where b is a charge- e boson and ψ_+, ψ_- are composite fermions. b and ψ_\pm carry charge 1 and -1 , respectively, under an internal emergent gauge field a , associated with the phase rotations $b \rightarrow e^{i\theta}b$, $\psi_\pm \rightarrow e^{-i\theta}\psi_\pm$ which keep the physical electron operator invariant. Furthermore, ψ_\pm carry charge $\pm 1/2$ under the external relative field A_r , which is associated with the transformations $c_\pm \rightarrow e^{\pm i\theta}c_\pm$, $\psi_\pm \rightarrow e^{\pm i\theta}\psi_\pm$.

Next, we assume a mean-field ansatz where b forms a bosonic $\nu = 1/2$ Laughlin state, and $\langle a \rangle = 0$. The resulting field theory can be written as

$$\mathcal{L} = -\frac{2}{4\pi}\tilde{a}\partial\tilde{a} + \frac{1}{2\pi}(a + A)\partial\tilde{a} + \mathcal{L}_\psi(\psi_\pm, a, A_r). \quad (1)$$

Here $a\partial a \equiv \epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$, $\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu\tilde{a}_\lambda$ is the conserved current for the b particles, and the first term on the RHS above is the effective action for a bosonic $1/2$ Laughlin FQH state:

[22]

$$\mathcal{L}_\psi = \sum_{\alpha=\pm} [\psi_\alpha^\dagger(i\partial_t + a_t + \alpha A_{r;t}/2)\psi_\alpha + \frac{1}{2m_\alpha}\psi_\alpha^\dagger(i\partial_i + a_i + \alpha A_{r;i}/2)^2\psi_\alpha + \dots], \quad (2)$$

where \dots indicates higher order interactions among the composite fermions. Now we can consider a variety of mean-field states for the composite fermions ψ_\pm .

(1) *Two-component composite Fermi liquid*. Here, ψ_\pm both form a composite Fermi sea. This describes a CFL state with two Fermi surfaces, with Fermi wave vectors $k_{F\pm} = \sqrt{2\pi n_\pm}$, where n_\pm is the density of electrons in the two layers. This phase is most natural when $\nu_- \sim 1/2$.

(2) *Z_2 fractionalized exciton metal*. We can consider a state where species ψ_+ forms a paired state, $\langle\psi_+\psi_+\rangle \neq 0$, while ψ_- continues to form a Fermi surface with $k_{F-} = \sqrt{2\pi n_-}$. This breaks the emergent $U(1)$ gauge symmetry down to Z_2 , and the Higgs mechanism sets $a + A_r/2 = 0$. In the limit $n_- = 0$, we expect ψ_+ forms a $p_x + ip_y$ state since the system is described by a Moore-Read Pfaffian state in the top layer [34] [23].

As n_- is adiabatically increased, the system is described by a Pfaffian state in ψ_+ together with a Fermi sea of the ψ_- . Since we have locked $a = -A_r/2$, we see from (2) that ψ_- effectively becomes coupled only to A_r , with unit charge. Physically, this implies that ψ_- now carries a unit dipole moment perpendicular to the layers, and can thus be identified with the f-Exc.

However, ψ_- is still coupled to an emergent Z_2 gauge field, corresponding to the remnant of a after the pairing of the ψ_+ fermions, reminiscent of the 'orthogonal metal' phase.[24] Importantly, the ψ_+ and ψ_- fermions are both coupled to this Z_2 gauge field, so are non-trivially entangled. In particular, since the neutral fermion ψ_{NF} inside the f-Exc acquires a π phase upon fully encircling the non-Abelian charge $e/4$ quasiparticle, the f-Exc sees any localized $\pm e/4$ quasiparticles pinned to disorder potentials as sources of random π flux.

A model wave function for this state can be written as follows: $\Psi(\{z_i, w_a\}) = \mathcal{P}_{LLL}\psi_{f\text{-exc}}(\{\mathbf{r}_a\}) \prod_{a<b}(w_a - w_b)^2 \prod_{i,a}(z_i - w_a)^2 \text{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j}(z_i - z_j)^2$. Here z and w are the complex coordinates of the electrons in the top and bottom layers, respectively, with $w_a = r_{a;x} + ir_{a;y}$. $\psi_{f\text{-exc}}(\{\mathbf{r}_a\})$ is the wave function for the excitons, which can be taken to be in a Fermi sea. \mathcal{P}_{LLL} denotes projection to the lowest Landau level. While this wave function is written in the case where both layers are in the lowest Landau level, it can be transposed to the case where the Pf layer is in the first Landau level by replacing acting with the LL raising operator on each upper-layer electron: $\prod_i(\partial_{z_i} - \frac{\bar{z}_i}{4\ell_B^2})$.

(3) *Interlayer coherent FQH states: exciton condensates*. We can consider a state where both ψ_\pm composite fermions form a paired state, $\langle\psi_+\psi_+\rangle \neq 0$, $\langle\psi_-\psi_-\rangle \neq 0$, which breaks $U(1)_r$ and gives interlayer coherence. As a result these phases

have a goldstone mode and superfluid-like counterflow. We can further distinguish two cases:

(a) $\langle \psi_+ \psi_- \rangle = 0$. In this case, pairs of the f-Exc have condensed, implying that the interlayer $U(1)_r$ is spontaneously broken down to Z_2 . This leaves behind a mod 2 conservation law for the exciton number. Since pairs of f-Exc are topologically equivalent to pairs of b-Exc, this state can also be viewed as a state where pairs of b-Exc have condensed.

(b) $\langle \psi_+ \psi_- \rangle \neq 0$. In this case, since we also have $\langle \psi_+ \psi_+ \rangle \neq 0$, we can treat $\langle \psi_+ \psi_- \rangle$ and $\langle \psi_+^\dagger \psi_- \rangle$ as equivalent. Since $\psi_+^\dagger \psi_-$ carries a unit $U(1)_r$ charge, its expectation value implies that the interlayer $U(1)_r$ is completely broken. This corresponds to the case where the b-Exc form a condensate.

In both case (a) and (b), we can further consider various types of paired states for the ψ_\pm fermions. Within mean-field theory, the ψ_\pm fermions are described by a Bogoliubov de Gennes Hamiltonian $H_{BdG}[\psi_\pm]$. The topological order of the resulting state is dictated by the Chern number associated with the ground state of $H_{BdG}[\psi_\pm]$. For example, if both ψ_\pm form a weak-pairing $p_x + ip_y$ BCS state in case (a), then the resulting topological order is that of the (331) FQH state, and the resulting state is a form of interlayer-coherent (331) state. If ψ_+ forms a weak-pairing $p_x + ip_y$ BCS state while ψ_- forms a strong-pairing $p_x + ip_y$ BCS state, again in case (a), then the result is an interlayer coherent Moore-Read Pfaffian state. These states are distinguished by how the coherence length compares to the inter-particle spacing. We expect the coherence length to be very weakly-dependent on n_- so, for sufficiently small n_- , we expect strong-pairing. For $n_- \approx n_+$, we expect both species of fermions to be in weak-pairing states, which gives the topological order of the 331 state.

Wave functions for these interlayer coherent FQH states can be written as $\Psi(\{x_i, \sigma_i\}) = \text{Pf}[g_{\sigma_i \sigma_j}(\mathbf{r}_i - \mathbf{r}_j)] \prod_{i < j} (x_i - x_j)^2$, where x_i is now the complex coordinate of the i th electron including both layers and $\sigma_i = \pm$ is its layer index. $g_{\sigma_i \sigma_j}(\mathbf{r}_i - \mathbf{r}_j)$ is the pair wave function. For example, if we take $g_{\sigma_i \sigma_j}(\mathbf{r}_i - \mathbf{r}_j) = \frac{\Delta_{\sigma \sigma'}}{x_i - x_j}$, this would correspond to the case where $\langle \psi_\sigma(k) \psi_{\sigma'}(-k) \rangle = \Delta_{\sigma \sigma'}(k_x + ik_y)$.

Since we are considering the case where the top and bottom layers correspond to partially filled $N = 1$ and $N = 0$ LL orbitals, which have different angular momentum quantum numbers, a state with non-zero amplitude for $g_{+-}(\mathbf{r} - \mathbf{r}')$ also spontaneously breaks rotational symmetry, in addition to the $U(1)_r$ symmetry.

(4) *Pfaffian FQH states with localized excitons*. Finally, we can consider a state where ψ_+ is paired, while the ψ_- fermions form a density wave state, or, in the presence of disorder, are localized. This is the state which, in the language of excitons used earlier, corresponds to a Pfaffian FQH state in one layer, in the presence of localized excitons. As explained above, this disordered state is not a sharply distinct phase from the Z_2 fractionalized exciton metal, but rather a different regime of the same phase. The topological order of such a state is simply that of the Pfaffian FQH state, regard-

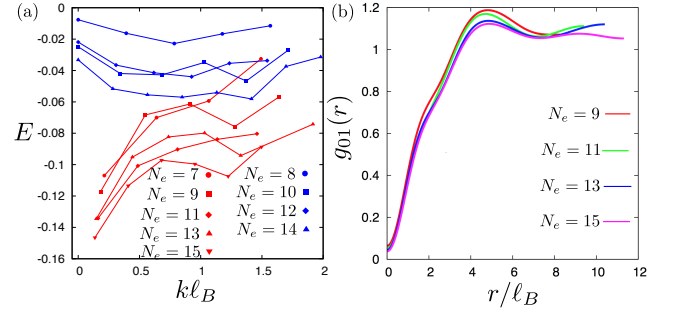


FIG. 1: (a) Bosonic and fermionic exciton energies as a function of momentum k , for a $N = 1$ PF-state with one particle excited to $N = 0$ level. The bosonic exciton (blue, $N_e = 2m$) is considerably higher in energy than the fermionic exciton (red, $N_e = 2m + 1$). Calculations are done for the Coulomb interaction, as described in the main text, with energies expressed in units of $e^2/\epsilon\ell_B$. (b) Pair correlation function between the $N = 0$ and $N = 1$ components in the f-Exc state. The hole in the $N = 1$ layer is bound to the electron in the $N = 0$ layer within a distance $\sim 4\ell_B$.

less of whether the b-Exc or f-Exc are lower in energy.

$(N_+, N_-) = (0, 0)$: (331) fractional exciton metal. In QH bilayers with $d/\ell_B > 1$, at filling $(\nu_+, \nu_-) = (1/4, 1/4)$ it is reasonable for the bilayer state to form a (331) state [35]. This would certainly be expected when both partially filled LLs are $N = 0$, although it may also happen more generally. When $(\nu_+, \nu_-) = (1/2, 0)$ or $(0, 1/2)$ and $(N_+, N_-) = (0, 0)$, the system will form a composite Fermi liquid state. What is the fate of the system in the intermediate regime, at $(\nu_+, \nu_-) = (1/4 + \delta, 1/4 - \delta)$? The (331) state also possesses an f-Exc, which contains charge $e/2$ and $-e/2$ in the two layers, respectively [36]. Note that this is quite distinct from the case considered above, where the f-Exc in the Pfaffian state contained charge e and $-e$ in the two layers. Since the b-Exc has charge e and $-e$ while the f-Exc has charge $e/2$ and $-e/2$, we expect that the Coulomb repulsion would cause the b-Exc to be unstable to decaying into two f-Exc's. As δ is tuned away from zero, the finite density of f-Exc's can form a Fermi sea. In terms of the effective theory, this can be described by writing $c_\pm = b_\pm \psi$, and assuming a mean-field state where b_\pm each form $\nu = 1/2$ bosonic Laughlin states in each layer, and ψ forms a $\nu = 1$ IQH state. The exciton in this picture can be thought of as a pair of $\pm e/2$ quasiparticles of the bosonic Laughlin states in each layer. As in the case of the Pfaffian exciton metal, the 331 state also possesses charge $\pm e/4$ quasiparticles, which the f-Exc's see as sources of an effective π -flux.

Exact diagonalization study of Pfaffian exciton energies. To investigate whether the b-Exc or f-Exc exciton has lower energy, we perform exact diagonalization of the Coulomb Hamiltonian on a sphere with both an $N = 0$ and $N = 1$ LL. Let us recall some facts about the Pfaffian state on a sphere. The Pfaffian ground state occurs when the number of electrons N_e is related to the number of LL orbitals N_{orb} by $N_{\text{orb}} = 2N_e - 2$. When N_e is even, the sphere has a unique

gapped ground state. However, when N_e is odd, there is a band of low energy states that disperses with k before merging into the continuum.[25, 26] This can be understood by appealing to the “superconducting” nature of Pfaffian phase [23]: when the number of composite fermions is odd, a composite fermion must appear as an unpaired BdG quasiparticle. This particle is precisely the neutral fermion ψ_{NF} excitation of the Pfaffian phase. By measuring the energy differences $E(N_e) - e_0 N_e$, where N_e is odd and e_0 is the energy per electron in the thermodynamic limit, one can obtain the energy of the neutral fermion, which was estimated to be $\Delta_{\text{NF}} \sim 0.15 - 0.028$ (all energies are given in terms of the Coulomb scale $e^2/\epsilon\ell_B$).

In a similar spirit, we use the difference between the ground state energies of the sphere for N_e -odd and even to probe the energy difference between the f-Exc and b-Exc. Let $E(N_1, N_0)$ be the energy with N_1, N_0 electrons in the $N = 1, 0$ levels respectively, keeping fixed $N_e = N_0 + N_1$ and the number of orbitals per component $N_{\text{orb}} = 2N_e - 2$. [37] We define the b-Exc energy $\Delta_{\text{b-Exc}}$ and f-Exc energy $\Delta_{\text{f-Exc}}$ by

$$E(N_e - 1, 1) - E(N_e, 0) = \Delta_{\text{b-Exc}} \quad (N_e \text{ even}) \quad (3)$$

$$E(N_e - 1, 1) - E(N_e, 0) = \Delta_{\text{f-Exc}} - \Delta_{\text{NF}} \quad (N_e \text{ odd}) \quad (4)$$

Note that for N_e -odd, $E(N_e, 0)$ contains both the vacuum energy and a neutral fermion, while $E(N_e - 1, 1)$ is the vacuum energy and a f-Exc, hence we subtract Δ_{NF} in the definition of $\Delta_{\text{f-Exc}}$. Also, if $\Delta_{\text{b-Exc}} > \Delta_{\text{f-Exc}} + \Delta_{\text{NF}}$, the bosonic exciton is not even a stable excitation, since it will decay. In this case, $\Delta_{\text{b-Exc}}$ should instead be interpreted as the energy of a f-Exc - ψ_{NF} pair.

In Fig. 1(a), we show the excitation spectra $\Delta_{\text{b-Exc}}(k)$ and $[\Delta_{\text{f-Exc}} - \Delta_{\text{NF}}](k)$. We obtain the lowest energy $E(N_e - 1, 1; L)$ in each of the angular momentum sectors L , which can be converted to linear momentum via $k\ell_B = L/\sqrt{(N_{\text{orb}} - 1)/2}$, while $E(N_e, 0)$ is obtained from the absolute ground state. The minima of the f-Exc sector is significantly lower in energy than the b-Exc sector, $(\Delta_{\text{f-Exc}} - \Delta_{\text{NF}}) - \Delta_{\text{b-Exc}} \sim -0.1$. Since previous studies report $2\Delta_{\text{NF}} < 0.1$, we interpret this to mean the b-Exc is unstable to decay to the f-Exc and a ψ_{NF} . This would also explain why the f-Exc has its minimum near $k = 0$, while the b-Exc minimum is near $k \sim 1$. If the b-Exc is in fact a composite of an f-Exc at $k = 0$ and a ψ_{NF} , the latter is known to have its minima near $k \sim 1$. [26] Unfortunately, finite size effects render it too difficult to estimate the effective mass $m_{\text{f-Exc}}$ from the present data. A comparison between the effective mass and effective interaction [27] would give important insight into the stability of the f-Exc Fermi surface.

To estimate the size of the exciton we examine the interlayer pair correlation function, Fig. 1(b). The ground state of the f-Exc occurs at angular momentum $L = \frac{1}{2}$, therefore it is two-fold degenerate, $\psi_{L_z = \pm \frac{1}{2}}$. We define the pair correlation function between the 0, 1 layers as $g_{01}(r) \equiv \frac{1}{2} \sum_{L_z = \pm \frac{1}{2}} \langle \psi_{L_z} | \hat{n}_1(0) \hat{n}_0(r) | \psi_{L_z} \rangle$, rescaled by the (uniform) electron density in each layer so that it approaches unity at

large distances. $1 - g_{01}(r)$ can be interpreted as the probability for the electron and hole to be at distance r , indicating they are bound within $\sim 4\ell_B$.

Experimental consequences.

Polarizability – The polarizability of the system is defined as $\lim_{\omega \rightarrow 0, q \rightarrow 0} \langle p(q, \omega) p(-q, -\omega) \rangle$, where $p(x, t) = n_+(x, t) - n_-(x, t)$ is the difference in density between the two components. All the states considered above have a finite polarizability. When the excitons are localized by disorder in either the bosonic or fermionic case, the polarizability is set by the disorder strength; in the Bose exciton condensate state it is set by the superfluid density, and in the exciton Fermi sea it is set by the density of states at the Fermi surface. The latter can be understood within the field theory presented above: if $\langle \psi_+ \psi_+ \rangle \neq 0$, then ψ_- is a f-Exc, $p \sim \psi_-^\dagger \psi_- + \text{const}$, and polarizability is simply the compressibility of the f-Exc state. The exciton Fermi sea can be differentiated by application of a periodic potential: when the wave vector of the periodic potential becomes commensurate with $2k_F$, Bragg scattering will induce a band gap for the excitons and modulate the polarizability.

Counterflow transport – Counterflow transport is a clear distinguishing feature of the three phases: localized Bose/Fermi excitons, interlayer coherent FQH states, and the exciton metal. Assuming the ability to independently contact the two different layers, one can measure the counterflow conductivity: $j_r = \sigma_r E_r$, where $j_r = j_+ - j_-$ is the relative current between the two layers, and $E_r = E_+ - E_-$ is the difference in electric field between the two layers. When $\langle \psi_+ \psi_+ \rangle \neq 0$, j_r is simply the current of the ψ_- fermions. The DC “counterflow conductivity” σ_r will thus be zero, finite, or infinite, depending on whether the excitons have Bose condensed, the f-excitons form a Fermi sea (and temperature T is greater than the localization cross-over scale), or the excitons are localized. The existence of a *dissipative* counterflow conductivity, in an incompressible FQH insulator, is a striking property of the exciton metal states.

Specific heat and thermal conductivity – Another characteristic distinguishing feature of the different exciton states appears in the specific heat and the thermal conductivity. The thermal conductivity of the exciton metal will be linear in temperature: $\kappa \sim C_v v_F \ell \sim T$, where ℓ is the mean free path of the excitons, v_F is their Fermi velocity, and $C_v \sim T$ is the specific heat of the exciton Fermi surface. Since such a state has zero electrical conductivity at zero temperature, this would imply an infinite violation of the Wiedemann-Franz law. In contrast, the thermal conductivity of the exciton localized state $\kappa \rightarrow 0$ at zero temperature, although the specific heat is still expected to be linear in T in this phase.

Umklapp process in BLG. In BLG, the $U(1)_r$ symmetry arises because the two components are in valleys with momenta $K \neq K'$, so scattering of a single particle between the components is forbidden by lattice translation and C_3 symmetry. In principle, however, there is an umklapp process in which three electrons are scattered from K to K' , since $3(K - K')$ is a reciprocal vector, breaking $U(1)_r$ down to

Z_3 . [28] Such a process is a ‘valley-anisotropy’ which weakly breaks the valley $SU(2)$ symmetry, and is suppressed by powers of $a/\ell_B \ll 1$, where a is the BLG lattice constant. While other valley-anisotropies are known to play a role in BLG, leading for instance to canted-antiferromagnetism at $\nu = 0$, [29–31] to our knowledge they all preserve $U(1)_r$. Thus it appears consistent to assume this umklapp process is very small, though a more detailed microscopic study would be helpful to determine what role it plays.

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 - [33] All even-denominator FQH states must contain a charge $-e$ boson, and thus will support a f-Exc.
 - [34] More generally, when ψ_+ forms any BCS paired odd angular momentum state, then other variants of the Pfaffian state are realized.
 - [35] The 331 state may potentially also occur when $d/\ell_B < 1$, although it is not well-understood when this can happen
 - [36] The transition between the Moore-Read and (331) can be viewed as a Dirac fermion transition, where the mass changes sign [32].
 - [37] Due to the different ‘shifts’ of the $N = 0, 1$ LLs, at fixed B -field the two components should have a different N_{orb} . However, we artificially adjust the monopole strength of the $N = 1$ level so that both have the same N_{orb} . To further reduce finite-size effects, we also use interaction pseudopotentials evaluated in the infinite plane.